Acta Technica 62, No. 1A/2017, 135–144

Application of quasi Monte Carlo polymerization re-sampling particle filter algorithm in airborne passive location[1]

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Abstract. Aiming at the problems that the the performance of the airborne passive location filter is poor and the noise is small, a method of Quasi Monte Carlo polymerization re-sampling particle filter algorithm is proposed in this paper, and which is applied to the airborne passive location. Weighted aggregation of similar particles in the discrete space is carried out, so as to make the particles in a reasonable space distribution, and also effectively suppress the degradation of the particles. Then the Quasi Monte Carlo technique is used to move the heavily sampled particles to the high likelihood region to optimize the distribution characteristics of the particles, and improve the accuracy of the filter. Finally, the simulation analysis of several algorithms is carried out in three conditions. The results show that the application of the Quasi Monte Carlo polymerization re-sampling particle filter algorithm in airborne passive location can improve the filtering precision and positioning efficiency.

Key words. Airborne passive location, quasi Monte Carlo, aggregated re-sampling particle filter.

1. Introduction

In the information warfare, because the airborne location radar has many problems [1], the research scholars have to search for the new airborne positioning technology at home and abroad, so that the passive location technology has been widely concerned [2]. Passive location technology has very important significance for the modern information warfare because of its low weight, wide range of positioning and strong concealment [3]. Passive tracking algorithm has become the core of passive location technology in recent years, because it cannot get the distance between the

[1]This work was supported by the Specialized Research Fund for the Doctoral Program of Higher Education of China under Grants No. 20130185110023.

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observation station and the signal transmitting point, so it needs to calculate the distance by the positioning algorithm [4]. In general, the distance measurement formula of the single observer passive location system is a very difficult nonlinear equation, at the same time, it has a large error in the position information, based on this, which must be processed by the observation equation of the passive location system. Compared with the traditional positioning and tracking algorithm, the single observer passive location is more difficult to be processed because of its own characteristics [5]. Because the observability performance of the system is weak, the equation is difficult to be solved, and the error is big, it also can appear the filter efficiency low and other problems, so the passive tracking algorithm has become the core of the research of the airborne positioning technology in recent years.

In recent years, the particle filter (PF) algorithm is applied to the nonlinear filtering problem, the probability distribution of random variables is calculated by a large number of random samples and their corresponding assignment. However, due to the poor performance of the airborne passive location, the initial errors and the co-variance of which are large. The standard particle filter algorithm is prone to degradation and impoverishment and other issues, which will lead to the poor filtering performance. In this regard, Klaas proposed a higher efficiency of the Gauss particle filter algorithm [6]. The core idea of which is to make the post probability distribution of airborne location state information approach to the Gauss distribution, and take the Quasi Monte Carlo integral to reduce the mean and co-variance of the sample. Gauss particle filter algorithm has no re-sampling, so the filter performance can be improved. However, in the use of single station passive location system, the noise is small, so it is easy to make data samples appear aggregation condition, so as to reduce the accuracy of positioning estimation. Based on this, Moradkhani proposed the Quasi Monte Carlo (QMC) algorithm and the Quasi Monte Carlo Goss particle filter (QMCGPF) algorithm, and the estimation accuracy could be obtained by using the Monte Carlo samples randomly generated in the sample space [7]. But because the operation rate is proportional to the number of the sample particles, so the speed of the operation of the system will be greatly decreased with the increase of the number of the samples.

Based on this, a method of Quasi Monte Carlo polymerization re-sampling particle filter algorithm is proposed in this paper, and which is applied to the airborne passive location. Weighted aggregation of similar particles in the discrete space is carried out, so that the particles are in a reasonable space distribution, so as to effectively suppress the degradation of the particles; The Quasi Monte Carlo technique is used to move the heavily sampled particles into the high likelihood region to optimize the distribution characteristics of the particles, so as to improve the accuracy of the filter. Finally, a variety of algorithms are simulated and analyzed.

2. State of the art

The airborne passive location system is quite different from the ground fixed position system, because its measurement is carried out under the coordinate system of the body, so it is necessary to convert the data measured by the airborne positioning system to the data of the ground fixed coordinate system. First of all, the body coordinate system is defined, it is assumed that the body centroid is the origin of the airborne passive location coordinate system, and the flight direction of the aircraft is the Y' axis, the direction which is perpendicular to the direction of the Y' axis is the Z' axis. Transformation of the coordinate system is shown in Fig. 1.

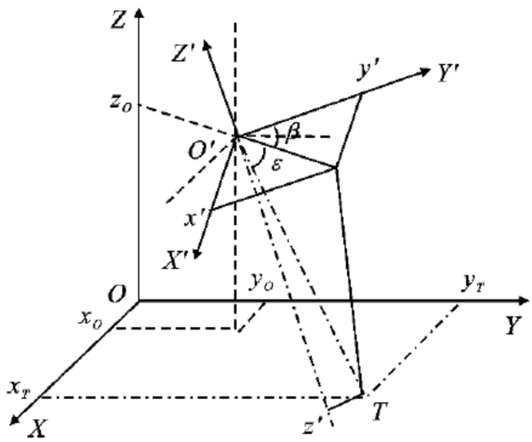

Fig. 1. Observer and object in three-dimensional geometry

The state vector and attitude information of the body at time T_k can be obtained by GPS and airborne navigation equipment. The relative state vector of the airborne passive location and the passive location in the body coordinate system is

$$X_{0k} = [x_{\mathrm{T}k},\, y_{\mathrm{T}k},\, z_{\mathrm{T}k},\, \dot{x}_{\mathrm{T}k},\, \dot{y}_{\mathrm{T}k},\, \dot{z}_{\mathrm{T}k}]^{\mathrm{T}} \,.$$

Here, $x_{\mathrm{T}k}$, $y_{\mathrm{T}k}$, $z_{\mathrm{T}k}$ is the target position in the XYZ coordinate system and $\dot{x}_{\mathrm{T}k}$, $\dot{y}_{\mathrm{T}k}$, $\dot{z}_{\mathrm{T}k}$ is the target position in the $X'Y'Z'$ coordinate system.

Based on the above assumptions, the coordinate conversion of the position vector can be considered as:

$$\left[x_k^{'},\, y_k^{'},\, z_k^{'}\right]^{\mathrm{T}} = A_k \left[x_{\mathrm{T}k} - x_{0k},\, y_{\mathrm{T}k} - y_{0k},\, -z_{0k}\right]^{\mathrm{T}} \,. \tag{1}$$

Here, A_k represents the matrix of the ground coordinate system and body coordinate system transformation [8], and x_{0k}, y_{0k}, z_{0k} is the position of point $0'$ in the XYZ coordinate system. Vector $\left[x_k^{'},\, y_k^{'},\, z_k^{'}\right]$ is speed. In the same way, the speed loss between the two systems can be expressed in the formula (1).

The state equation and observation equation are established in the following way:

$$X_{Tk+1} = f\left(X_{Tk}, w_k\right) = \Phi_k X_{Tk} + G_k w_k \,, \tag{2}$$

$$Z_k = \left[\beta_k, \varepsilon_k, \dot{\beta}_k, \dot{\varepsilon}_k, \dot{f}_{dk}\right]^{\mathrm{T}} + v_k \,. \tag{3}$$

In the equation, the state transition matrix is represented by

$$\Phi_k = \begin{bmatrix} I_2 & TI_2 \\ 0 & I_2 \end{bmatrix}, \quad G_k = \begin{bmatrix} T^2 I_2/2 \\ TI_2 \end{bmatrix}.$$

The process noise is expressed by w_k, and the observation noise is expressed by v_k. The process noise and observation noise are independent of each other, and the Gauss noise is of zero mean; $E\left[w_i, w_j^{\mathrm{T}}\right] = O_k\delta_{ij}$, $E\left[v_i, v_j^T\right] = R_k\delta_{ij}$, the measurement period is expressed by T, and the 2-order unit matrix is expressed by I_2. Symbols X_{Tk+1} and X_{Tk} are $k+1$th and kth time state vectors, and Z_k is kth time observation vector. Symbols β_k and ε_k are target bear in the XYZ coordinates in the kth time while $\dot{\beta}_k$ and $\dot{\varepsilon}_k$ represent target bear in the $X'Y'Z'$ coordinates in the kth time. Finally, $\dot{f}_{dk}$ is frequency offset in $k-1$th and kth time instants.

Based on the principles of particle kinematics, the following equations are used to express the observed variables:

$$\beta_k = \arctan\left(x_k^{'}/y_k^{'}\right) \,, \tag{4}$$

$$\varepsilon_k = \arctan\left(z_k^{'}/\left(x_k^{'2} + y_k^{'2}\right)^{1/2} \,,\right) \tag{5}$$

$$\dot{\beta}_k = \left(y_k^{'}\dot{x}_k^{'} - x_k^{'}\dot{y}_k^{'}\right) / \left(x_k^{'2} + y_k^{'2}\right) \,, \tag{6}$$

$$\begin{aligned} \dot{\varepsilon}_k = & \left[-x_k^{'}z_k^{'}\dot{x}_k^{'} - y_k^{'}z_k^{'}\dot{y}_k^{'} + \left(x_k^{'2} + y_k^{'2}\right)\dot{z}_k^{'}\right] / \\ & / \left[\left(x_k^{'2} + y_k^{'2}\right)^{1/2}\left(x_k^{'2} + y_k^{'2} + z_k^{'2}\right)\right] .\,, \end{aligned} \tag{7}$$

where $\dot{x}_k^{'}$, $\dot{y}_k^{'}$, $\dot{z}_k^{'}$ are changes of the velocity components. When a relative radial velocity appears between the observation station and the aircraft radio frequency, the Doppler frequency will be received by the observation station $f = f_T + f_d$. The value of f_T (frequency of aircraft RF) is set to a constant amount, then f_d is used to represent the Doppler frequency, and the formula is as follows

$$\begin{aligned} f_{dk} = & -f_T(\dot{x}_k^{'} \sin\beta_k \cos\varepsilon_k + \dot{y}_k^{'} \cos\beta_k \cos\varepsilon_k + \\ & + \dot{z}_k^{'} \sin\varepsilon_k)/c \,. \end{aligned} \tag{8}$$

The Doppler frequency change rate of the can be obtained:

$$\begin{aligned} \dot{f}_{dk} = & -f_T(\ddot{x}_k^{'} \sin\beta_k \cos\varepsilon_k + \ddot{y}_k^{'} \cos\beta_k \cos\varepsilon_k + \\ & + \ddot{z}_k^{'} \sin\varepsilon_k + r_k\left(\dot{\beta}_k \cos\varepsilon_k\right)^2 + r_k\dot{\varepsilon}_k^2)/c \,. \end{aligned} \tag{9}$$

In this expression, symbol c represents the propagation velocity of electromagnetic wave, while $\ddot{x}_k^{'}$, $\ddot{y}_k^{'}$, $\ddot{z}_k^{'}$ is the vector of acceleration.

When a relatively large maneuver is generated from the observation station and the aircraft, the acceleration term in (9) can be ignored, so that

$$\dot{f}_{dk} = -f_T \left[r_k \left(\dot{\beta}_k \cos \varepsilon_k \right)^2 + r_k \dot{\varepsilon}_k^2 \right] / c \,. \tag{10}$$

3. METHODOLOGY

3.1. Thompson-Taylor algorithm

As a new algorithm, Thompson-Taylor algorithm is mainly used in the generation of random samples [9]. The basic principle of the algorithm is to deal with the random number of samples which are similar to that of m samples by centralized processing, and then to get a new random sample of the m samples. Its advantage consists in the fact that it will not be too dependent on the distribution of the state space of the sample, which is not necessary to be similar to Gauss approximation. Its process is as follows:

Step one: a random sample x^i is extracted from the sample set $\left\{x^i\right\}_{i=1}^{N}$, and the adjacent m samples $\left\{x_1^i, x_2^i, \cdots, x_m^i\right\}$ (including x^i) are obtained by smooth operation, and the average value of the m samples is $\bar{x}^i$.

Step two: a random number set is obtained:

$$\{u_i\}_{i=1}^{m} \sim U \left(\frac{1}{m} - \sqrt{\frac{3m-3}{m^2}}, \frac{1}{m} + \sqrt{\frac{3m-3}{m^2}} \right) . \tag{11}$$

Step three: new random sample is generated:

$$z^i = \bar{x}^i + u_i \left(x_k^i - \bar{x}^i \right), k = 1, 2, \cdots, m \,. \tag{12}$$

Thompson-Taylor algorithm can generate a more uniform random sample distribution, and keep the the same mean and variance of the sample set, although it is not for the particle filter algorithm, but the method can be used to make the particle filter sample diversity to be met.

3.2. Quasi Monte Carlo re-sampling particle filter

Because the operation of the Quasi Monte Carlo Goss particle filter algorithm has a positive correlation with the number of Quasi Monte Carlo samples [10], so the decrease of the number of Quasi Monte Carlo samples can improve the computing speed, so as to obtain a new algorithm, that is, the Quasi Monte Carlo aggregated re-sampling particle filter algorithm [11]. Its core idea is: the weighted aggregation of the similar particles in the state space is carried out, the boundary conditions is set as the average forecast in the space of the center of the polymer particle, and the Quasi Monte Carlo re-sampling is carried out in the space. In this process, the Quasi

Monte Carlo sampling is carried out in the neighborhood of the aggregated particles, and the steps of the Quasi Monte Carlo re-sampling particle filter prediction sampling space are omitted, so that the diversity of the sample is improved, and the filtering precision and the positioning accuracy are improved, too [12].

Initialization: The initial relative distance $\hat{r}_0 = -\lambda \dot{f}_{d0} / \left(\dot{\varepsilon}_0^2 + \dot{\beta}_0^2 \cos^2 \varepsilon_0 \right)$ obtained by the initial observation is combined with the azimuth angle and pitching angle to obtain the observed object. Based on the estimation $\left[\hat{x}_0^{'}, \hat{y}_0^{'}, \hat{z}_0^{'} \right]$ of the position vector of the body coordinate system and the $\hat{X}_{O0}$, the two-dimensional position vector estimation $\left[\hat{x}_{T0}^{'}, \hat{y}_{T0}^{'} \right]^T$ of the observed object in the ground coordinate system is obtained, so as to get the initial state $\hat{X}_{T0}$, the initial error co-variance matrix $\hat{P}_0$ is calculated based on the initial observation error.

Quasi Monte Carlo sampling: Based on the example of the HALTON sequence, the method that can generate the Gauss point is given, so as to generate N Gauss points which obey the $P\left(x_{k-1}\right)$:

$$\left\{ x_{k-1}^{(i)} \right\}_{i=1}^{N} \sim N\left(x_{k-1}; \bar{x}_{k-1}, \hat{P}_{k-1} \right) . \tag{13}$$

The particle set $\left\{ x_{k|k-1}^{(i)} \right\}_{i=1}^{N}$ is predicted at the k moments according to the state equation,and the mean and co-variance of the $\left\{ x_{k|k-1}^{(i)} \right\}_{i=1}^{N}$ are estimated, that is:

$$\bar{x}_{\,k|k-1} = \frac{1}{N} \sum_{i=1}^{N} x_{k|k-1}^{(i)} , \tag{14}$$

$$\hat{P}_{k|k-1} = \frac{1}{N} \sum_{i=1}^{N} \left(x_{k|k-1}^{(i)} - \bar{x}_{\,k|k-1} \right) \left(x_{k|k-1}^{(i)} - x^{\,k|k-1} \right)^{\mathrm{T}} . \tag{15}$$

According to the importance density $q\left(x_k \left| z_{1,k} \right.\right)$, the quasi Gauss sample $\left\{ x_k^{(i)} \right\}_{i=1}^{N}$ is extracted by the Quasi Monte Carlo sampling.

According to the observed value Z_{k}, the weight value $\omega_k^{(i)}$ of each particle is calculated and its normalization processing is carried out, that is

$$\omega_k^{(i)} = P\left(z_k \left| x_k^{(i)} \right. \right) N\left(x_k^{(i)}; \bar{x}_{\,k|k-1}, \hat{P}_{k|k-1} \right) / q\left(x_k \left| z_{1,k} \right. \right) \omega_k^{(i)} = \omega_k^{(i)} / \sum_{i=1}^{N} \omega_k^{(i)} . \tag{16}$$

The state of the target and the posterior distribution after the K moments are estimated:

$$\bar{x}_k = \sum_{i=1}^{N} \omega_k^{(i)} x_k^{(i)}, \quad \hat{P}_k = \sum_{i=1}^{N} \omega_k^{(i)} \left(x_k^{(i)} - \bar{x}_k \right) \left(x_k^{(i)} - \bar{x}_k \right)^{\mathrm{T}} . \tag{17}$$

Particle aggregation: the number of particles $\#N_{ki}^2,\ i=1,2,\cdots,m$ in each grid set $\#G_{ki}^2$ is recorded, among them

$$\#N_{k1}^2+\#N_{k2}^2+\cdots+\#N_{km}^2=N.$$

. Then the particles in each grid are focused, so as to get the m polymer particles $\left\{\left(x_k^{t_i},\omega_k^{t_i}\right)\right\}_{i=1}^{m}$.

Quasi Monte Carlo particle aggregation re-sampling: A four dimensional random Quasi Monte Carlo sequence $\{u_i\}_{i=1}^{N-m}$ is truncated by the particle, so as to get m sub-sequences with the length of $\#N_{ki}^2-1$, then:

$$x_k^{t_i}(j)=x_k^{t_i}+\left(x_k^{t_i}-\bar{x}_k\right)\times u_j\,. \tag{18}$$

Based on the above formula, $\#N_{ki}^2-1$ Quasi Monte Carlo sampling particle $\left\{\left(x_k^{t_i}(j),\omega_k^{t_i}(j)\right)\right\}_{j=1}^{\#N_{ki}^2-1}$ is obtained. Finally, the average weight of all the particles in the grid is solved, and the formula is as follows:

$$\omega_k^{t_i}(j)=\omega_k^{t_i}/\#N_{ki}^2\,. \tag{19}$$

4. Result analysis and discussion

In this paper, three sets of experiments are set up, under different observation conditions of the Quasi Monte Carlo re-sampling particle filter algorithm. The comparison of the performance of the particle filter algorithm, Gauss particle filter algorithm and the Quasi Monte Carlo Goss particle filter algorithm is carried out. Among them, the particle filter algorithm uses re-sampling, the simulation parameters are set as follows: it is assumed that in the ground coordinates, the starting position of the machine is (0,2.0 km), the navigation speed of the aircraft is (20 m/s, 0, 0), the accuracy of airborne equipment is $\sigma_{x_0}=\sigma_{y_0}=20\,\mathrm{m}$, $\sigma_{z_0}=8\,\mathrm{m}$; the initial position of the signal receiving station is (160 km, 160 km), the absolute velocity is (-15 m/s, 15 m/s). The measurement accuracy of the three groups is summarized in the following table:

experiment 1	$\sigma_\beta=\sigma_\varepsilon=1.64\times10^{-3}\,\mathrm{rad}$	$\sigma_{\dot{\mathrm{f}}_\mathrm{d}}=1\,\mathrm{Hz}$	$\sigma_{\dot\beta}=\sigma_{\dot\varepsilon}=0.1\times10^{-3}\,\mathrm{rad/s}$
experiment 2	$\sigma_\beta=\sigma_\varepsilon=28.4\times10^{-3}\,\mathrm{rad}$	$\sigma_{\dot{\mathrm{f}}_\mathrm{d}}=2\,\mathrm{Hz}$	$\sigma_{\dot\beta}=\sigma_{\dot\varepsilon}=0.2\times10^{-3}\,\mathrm{rad/s}$
experiment 3	$\sigma_\beta=\sigma_\varepsilon=35.7\times10^{-3}\,\mathrm{rad}$	$\sigma_{\dot{\mathrm{f}}_\mathrm{d}}=4\,\mathrm{Hz}$	$\sigma_{\dot\beta}=\sigma_{\dot\varepsilon}=0.3\times10^{-3}\,\mathrm{rad/s}$

In the three groups of experiments, $\sigma_{f_T}=10\,\mathrm{MHz}$, the sample period is 1 s, the number of observations is 100, and the number of selected particles $N=400$. The performance index of each algorithm adopts the relative distance error E_{rr}, and the degradation degree of the particle is expressed by the mean effective particle number

N_{eff}, that is:

$$E_{\text{rr}} = \frac{\left(x_{\text{ture}}^2 - \hat{x}\right)^2 + \left(y_{\text{ture}}^2 - \hat{y}\right)^2}{\left(x_{\text{ture}}^2 + y_{\text{ture}}^2\right)^2} \times 100\,\%\,, \tag{20}$$

$$N_{\text{eff},k} = 1/\sum_{i=1}^{N} \left(\omega_k^i\right)^2, \quad N_{\text{eff}} = \frac{1}{100}\sum_{k=1}^{100} N_{\text{eff},k}\,. \tag{21}$$

The simulation results are shown in Figs. 2, 3 and 4.

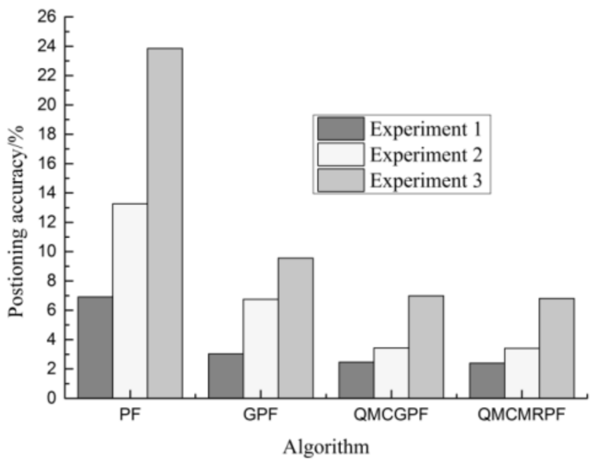

Fig. 2. Comparison of positioning accuracy in different observation accuracy

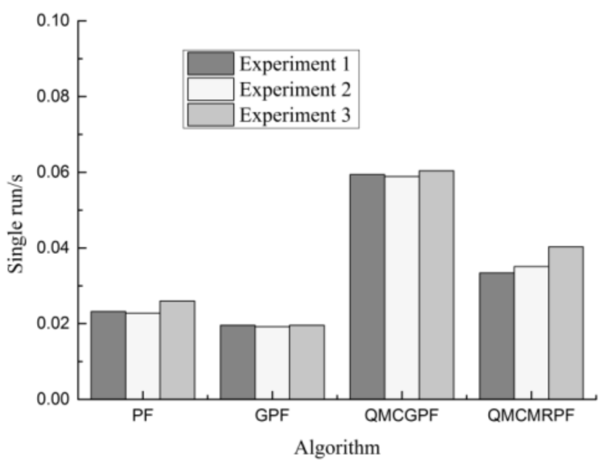

Fig. 3. Comparison of single operation time in different observation accuracy

From Figs. 2–4, it can be drawn that the observation accuracy is proportional to the positioning accuracy of the algorithm. The positioning accuracy of particle

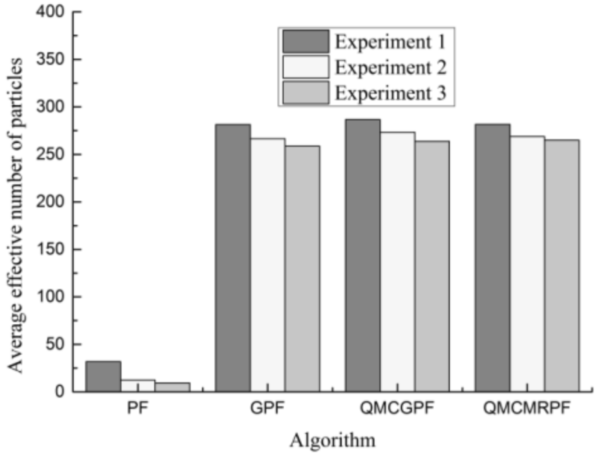

Fig. 4. Comparison of average effective number of particles in different observation accuracy

filter algorithm is approximate to a certain value, and will continue to maintain convergence. In addition, from the number of the average effective particle, it can seen that the particle filter algorithm exhibits serious particle degeneracy and impoverishment, while the Gauss particle filtering algorithm does not exhibit the phenomenon of impoverishment because of the Gauss distribution. The main reason is that Gauss's assumption is established, at the same time, the Gauss particle filter algorithm obtains the samples from the continuous distribution of the state space, and the Gauss interference of each sample is also carried out, so as to increase the diversity of particles, and avoid the emergence of the phenomenon of dilution, which shows that Gaussian particle filter algorithm is obviously superior to the particle filter algorithm. Quasi Monte Carlo Goss particle filter algorithm uses the Quasi Monte Carlo sampling on the basis of the Gaussian particle filter algorithm, so that the sample distribution in the state space is more uniform, so as to further improve the precision of estimation. Therefore, under the same conditions, the accuracy of the Quasi Monte Carlo Goss particle filter algorithm is much higher than that of the Gauss particle filter algorithm. In addition, the Quasi Monte Carl Goss particle filter algorithm improves the positioning accuracy, and the operation time of this algorithm is significantly improved.

5. Conclusion

In this paper, Quasi Monte Carlo aggregated re-sampling particle filter algorithm based on the airborne passive location method is proposed, and the following conclusions are obtained:

Through the simulation and analysis of a variety of filtering algorithms, Gaussian particle filter algorithm cannot appear phenomenon of impoverishment because of its Gaussian distribution, the main reason is that the Gauss particle filter algorithm carries out the Gauss interference for each sample, so as to increase the diversity of particles, and avoid the emergence of the phenomenon of dilution, so it can be known that the Gauss particle filter algorithm is significantly better than the particle filter algorithm. At the same time, Quasi Monte Carlo Goss particle filter algorithm uses the Quasi Monte Carlo sampling on the basis of the Gaussian particle filter algorithm, so that the sample distribution of the state space is more uniform, so as to further improve the precision of estimation.

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Received April 23, 2017